Engineering mechanics is the science which deals with physical state of rest or motion of bodies under the action of forces.

ENGINEERING MECHANICS
- STATICS (Body is at rest)
- DYNAMICS (Body is in motion)
  - KINEMATICS
  - KINETICS

<u>KINEMATICS</u> : Study of a body in motion, when the forces which cause the motion are not considered.

<u>KINETICS</u> : Study of a body in motion, when the forces which cause the motion are considered.

<u>RIGID BODIES</u> : These are bodies which donot deform under the action of applied forces.

<u>PARTICLE</u> : It is defined as an object whose mass is concentrated at a point.

FORCE →

It is an agency which changes or tends to change the position of rest or of uniform velocity/motion of a body.
A force can produce push, pull or twist in a body.

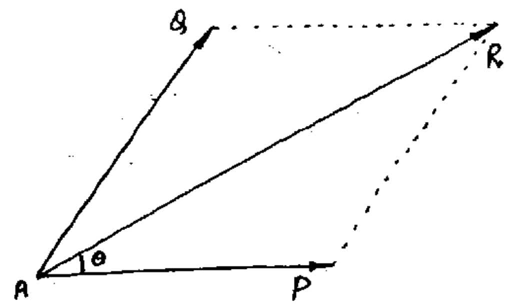
ADDITION OF TWO FORCES : PARALLELOGRAM LAW

1.) EQUAL FORCE → Two forces are equal only if they have same magnitude and direction even if they do not have same point of application.

2.) EQUIVALENT FORCE → Two forces are equivalent if they produce some specific effect on the rigid body.

PARALLELOGRAM LAW

" If two forces acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal passing through their point of intersection represer the resultant in both magnitude and direction."



Consider two forces P and Q acting at a point A. Now let these two forces be represented by the sides of a parallelogram.

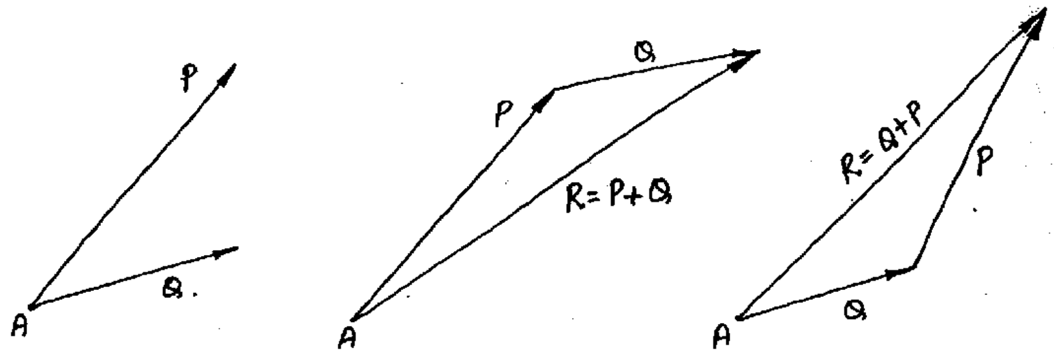
The diagonal passing through the point A represents the sum or the resultant of the forces P and Q.

$$\boxed{R = P + Q}$$

LAW OF TRIANGLE OF FORCES

"If two forces acting at a point are represented by the two sides of a triangle taken in order, then their sum or resultant is represented by the third side taken in an opposite order."

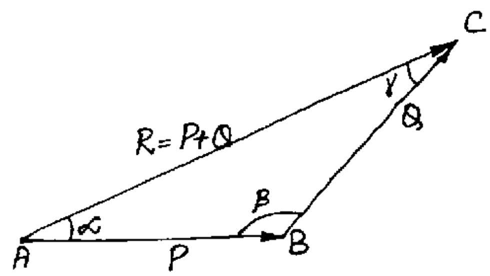
The sum or resultant of two forces is obtained by constructing a triangle such that forces represe the two sides taken in an order and the closing side is taken in opposite order representing the sum or resultant.



Sum / Resultant, $R = P + Q = Q + R$.

If included angle between the two forces is known, then resultant (magnitude) is calculated by,

$$\boxed{R^2 = P^2 + Q^2 - 2PQ\cos\beta}$$



Remaining angles can be calculated by law of sines

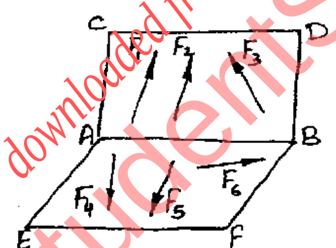
$$\frac{P}{\sin\gamma} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\beta}$$

CONCEPT OF RESULTANT OF SEVERAL FORCES

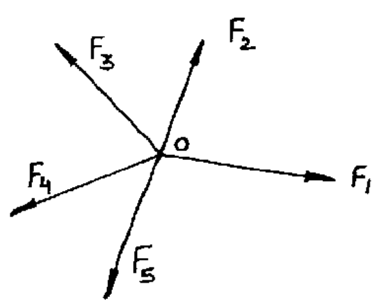
When a number of forces produce same effect on the rigid body as that produced by a single force then the single force is termed as the resultant of those several forces.

1.) **POINT FORCE** → A finite force transmitted through an infinitesimal area or a point is known as point force.

2.) **COPLANAR FORCES** → When a number of forces lie in the plane then they are known as coplanar forces.



3.) **CONCURRENT FORCES** → These are the forces whose lines of action pass through a common point.

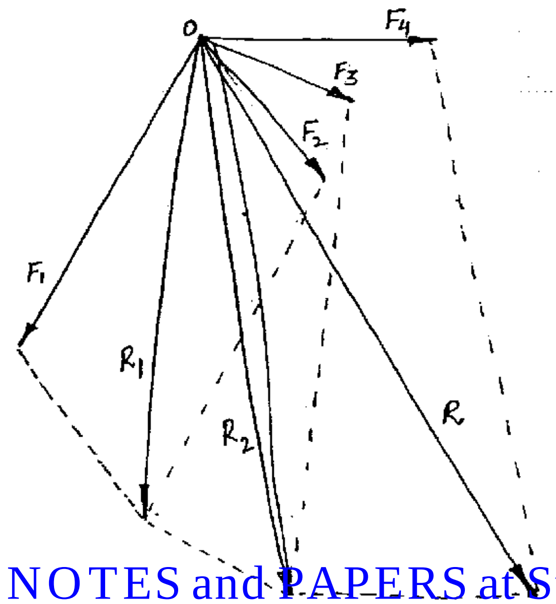


4.) PARALLEL FORCES → These forces have their lines of action parallel to each other.

5.) GENERAL SYSTEM OF FORCES → If no. of forces acting in a plane are such that they do not intersect in one point and are not parallel also.

RESULTANT OF SEVERAL CONCURRENT COPLANAR FORCES

If number of coplanar forces are acting at a point, then the resultant can be found by repeated use of parallelogram law.



Resultant of force F_1 and F_2 $= R_1$

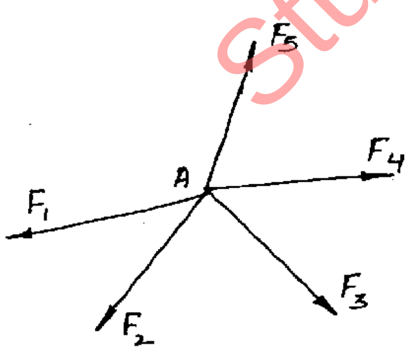
Resultant of Force F_3 and resultant of Force F_1 and F_2

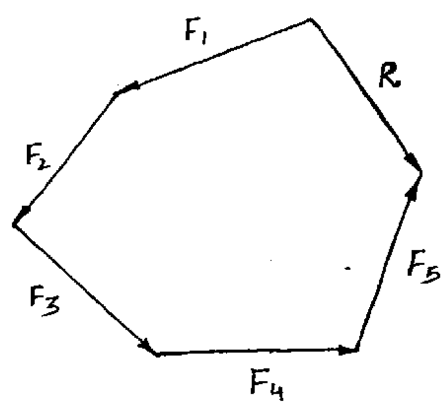
$$= F_3 + R_1 = R_2$$

Resultant of Force F_4 and Resultant of force F_3 and F_1 and F_2 $= F_4 + R_2 = R_3/R.$

LAW OF POLYGON OF FORCES

"If a number of coplanar forces are acting at a point such that they can be represented in magnitude and direction by the sides of a polygon taken in an order, their resultant is represented in both magnitude and direction by the closing side of the polygon taken in the opposite order."





Resultant does not depend on the order in which the forces are chosen to draw the polygon.

RESOLUTION OF A FORCE INTO COMPONENTS

In resolving the force F, a single force acting on a body is replaced by two or more forces which together have same effect as the single force. These forces are known as components of the original force F.

CASE 1:

One component is known in magnitude and direction and the other is to be determined.

This can be done graphically by

(1) Parallelogram Law
(2) Triangle Law

CASE 2:

When the lines of action of both components are known but their magnitudes are to be determined. (Parallelogram Law)

RESOLUTION INTO RECTANGULAR COMPONENTS

When a force F is resolved into components which are perpendicular to each other, such components are known as rectangular components.

Consider force F, resolved into two rectangular components along x and y axes.

On adding vectorially R_x and R_y

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta = \frac{R_y}{R_x}$$

EQUATIONS OF EQUILIBRIUM FOR A SYSTEM OF CONCURRENT FORCES IN A PLANE

If the resultant of number of forces is equal to zero, then the particle is said to be in equilibrium.

For $R = 0$

$R_x = 0$ and $R_y = 0$

$\therefore \quad R_x = F_{x_1} + F_{x_2} + F_{x_3} + \ldots = 0$

$$\boxed{\Sigma F_x = 0}$$

$R_y = F_{y_1} + F_{y_2} + F_{y_3} + \ldots = 0$

$$\boxed{\Sigma F_y = 0}$$

Hence the equations

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

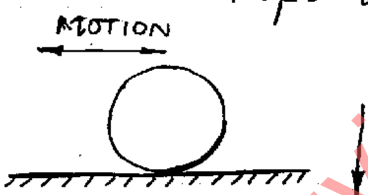
are known as the equations of equilibrium.

Concurrent forces lying in a plane are said to be in equilibrium if these equilibrium equations are satisfied.

CONSTRAINT, ACTION AND REACTION

CONSTRAINT : Restriction to the free motion of a body in any direction.

Example → A ball kept on a horizontal plane



Vertical downward movement is restricted by the horizontal plane

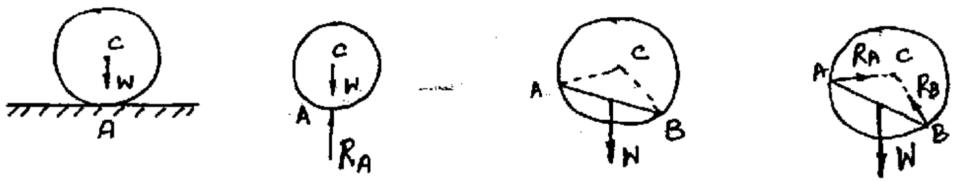
→ The action of a constrained body on any support induces an equal and opposite reaction from the support.

1) Linear motion restriction by support in any direction, the reaction would be a force in that direction.

2) Rotation is restricted about a point, the reaction is a couple about that point.

TYPES OF SUPPORT AND SUPPORT REACTIONS

1) **FRICTIONLESS SUPPORT :** Reaction acts normal to the surface at the point of contact.

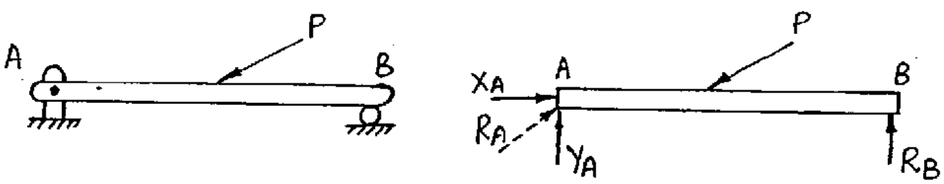


Sphere resting on a Horizontal plane

A rod resting inside a sphere.

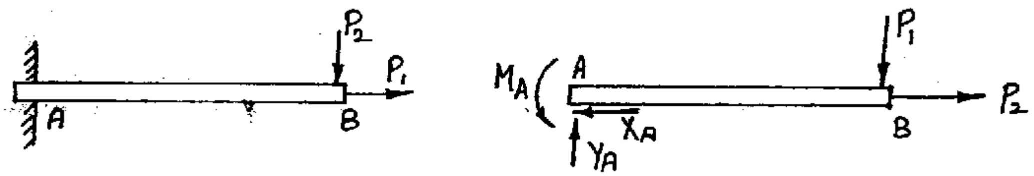
2.) ROLLER AND KNIFE EDGE SUPPORTS: These restrict the motion normal to the surface of the beam. In such supports, the reaction acts normal to the surface at points of contact.

3.) HINGED SUPPORT: It restricts the motion of the hinged end of the beam both in horizontal as well as vertical directions. Hence two independent reactions act at the hinged end. These reactions can be represented by single force in unknown direction



4.) BUILT-IN-SUPPORT: If a end of a beam is embed in concrete, it restricts the motion of end in horizontal and vertical directions along with restriction of rotation of beam about the point.

Therefore, two independent reactions X_A and Y_A act at the point along with reaction couple M_A.

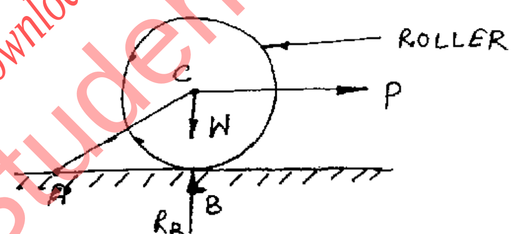


FREE-BODY DIAGRAM

In this, all the supports are replaced by the reactions which the supports exert on the body.

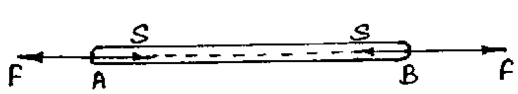
TWO TYPES OF FORCES

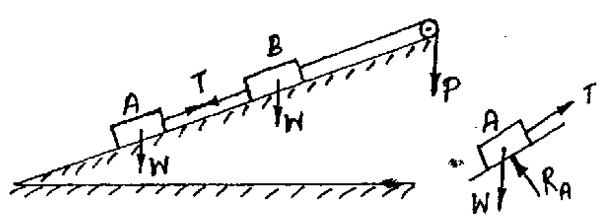
1) EXTERNAL FORCE ⟶ These forces act on a body or system of bodies from outside.



External forces are : Applied force P
Weight of roller W
Reaction at point of contact R_B

2) INTERNAL FORCE ⟶ These forces hold the particles of a body or two bodies together.



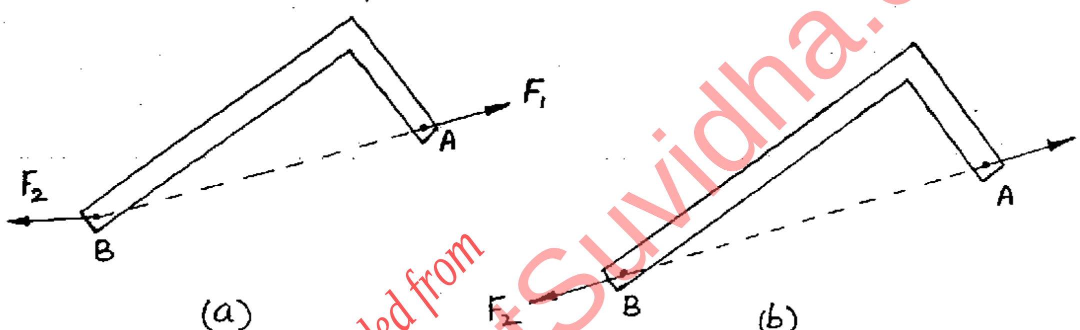


EQUILIBRIUM OF A BODY SUBJECTED TO TWO FORCES (8)

[TWO FORCE BODY]

A body subjected to forces acting only at the two points is known as two force body.

Let a L-shaped plate is acted upon by two forces F_1 and F_2 at the ends A and B.



Two forces can be in equilibrium only if they:

1) Are equal in magnitude
2) Opposite in direction
3) Have same line of action

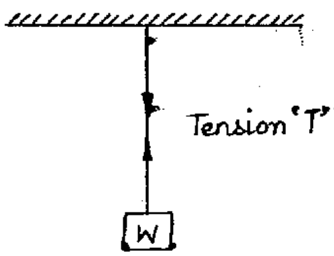
In figure (a) If $F_1 = F_2$, even then the body is not in equilibrium because they do not have same line of action.

In figure (b) The body is in equilibrium when $F_1 = F_2$.

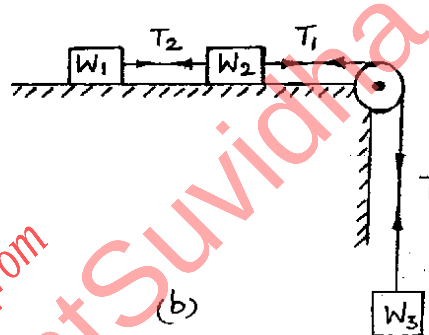
TENSION IN THE STRING, ROPE, BELT, CABLE AND CHAIN

Consider a body of weight W supported at the end of a string attached to a fixed support. Tension in the string is an internal force.

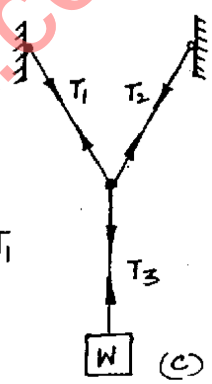
→ In a continuous string, rope, belt, cable or chain passing over a pulley, the tension remains same throughout.



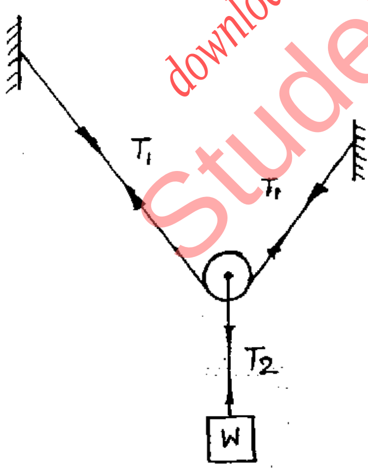
(a)



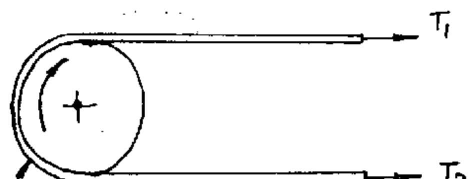
(b)



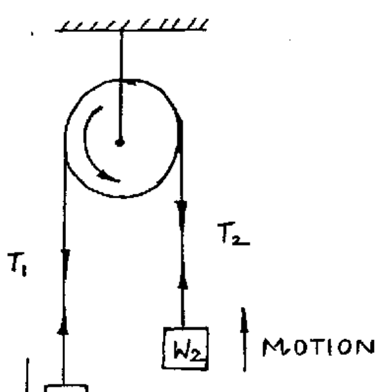
(c)



(d)



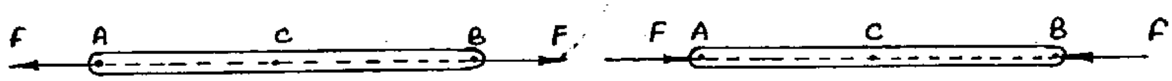
(e)



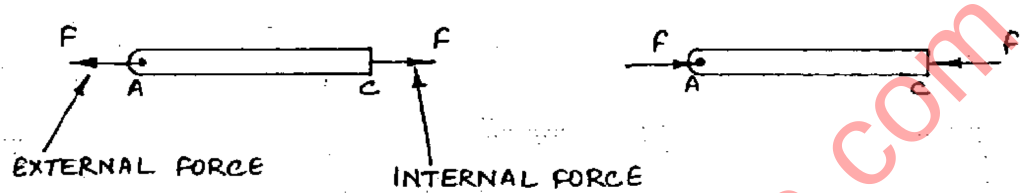
(f)

REPRESENTATION OF AXIAL FORCES IN A BAR

1) EXTERNAL FORCES →



TENSILE FORCES COMPRESSIVE FORCES



2) INTERNAL FORCES →

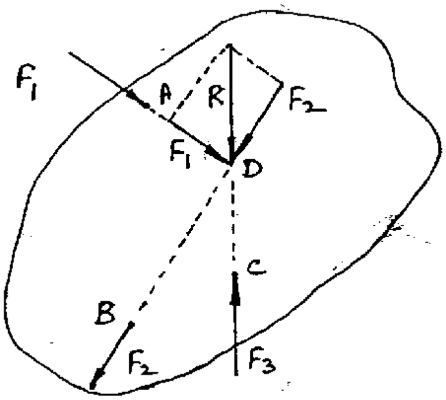
TENSILE FORCE COMPRESSIVE FORCES

EQUILIBRIUM OF A BODY SUBJECTED TO THREE FORCES

When a body is acted upon by three coplanar forces, it can be in equilibrium if

1) Their lines of action intersect at one point
2) They are parallel.

Consider three non-parallel forces F_1, F_2 and F_3 acting at points A, B and C.

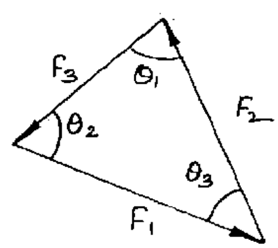


D is the point where the lines of action of forces F_1 and F_2 meet.

Force F_3 or the resultant will keep the body in equilibrium if it passes through the point D.

Hence,

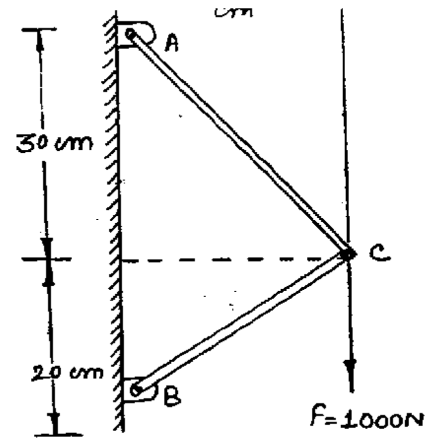
"Three concurrent forces in equilibrium must form a closed triangle of force when drawn in head to tail fashion."



Now, Law of sines can be used.

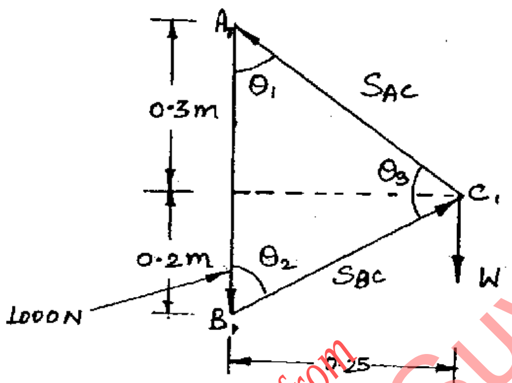
$$\boxed{\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}}$$

Q1.



Axial forces in bars AB and AC are internal forces.
External forces at C point be S_{AC} and S_{BC}.

<u>Sol</u>:



By law of Sines :-

$$\frac{S_{BC}}{BC} = \frac{S_{AC}}{AC} = \frac{1000}{AB}$$

[ΔABC is similar to $\Delta A_1B_1C_1$

$AB = 0{\cdot}5m$

$AC = \sqrt{(0{\cdot}3)^2 + (0{\cdot}25)^2}$

$= \sqrt{0{\cdot}09 + 0{\cdot}0625}$

$AC = 0{\cdot}3905\ m$

$BC = \sqrt{(0{\cdot}2)^2 + (0{\cdot}25)^2}$

$= \sqrt{0{\cdot}04 + 0{\cdot}0625}$

$BC = 0{\cdot}3201\ m$

$\sin\theta_1 = \frac{0{\cdot}25}{0{\cdot}3905} = 0{\cdot}64$

$\sin\theta_2 = \frac{0{\cdot}25}{0{\cdot}3201} = 0{\cdot}78$

$\theta_3 = [180° - (\sin^{-1}0{\cdot}64 + \sin^{-1}0{\cdot}78)]$

$= 180° - (39{\cdot}79° + 51{\cdot}26°)$

$\theta_3 = 88{\cdot}95$ [illegible]

Now,

$$\frac{S_{BC}}{\sin\theta_1} = \frac{S_{AC}}{\sin\theta_2} = \frac{AB}{\sin\theta_3}$$

$S_{BC} = 1000 \times \frac{\sin\theta_1}{\sin\theta_3}$

$= 1000 \times 0{\cdot}64$

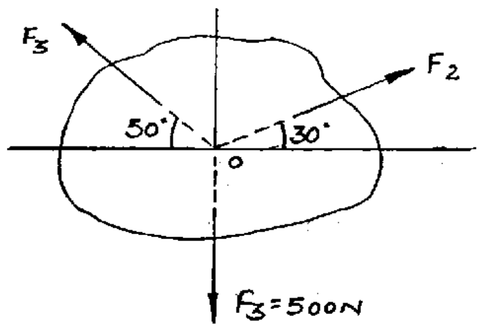
$\boxed{S_{BC} = 640N}$

$S_{AC} = 1000 \times \frac{\sin\theta_2}{\sin\theta_3}$

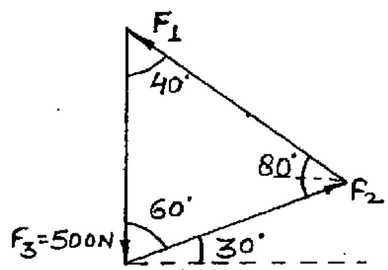
$= 1000 \times 0{\cdot}78$

$\boxed{S_{AC} = 780N}$

Q2.



SOL:



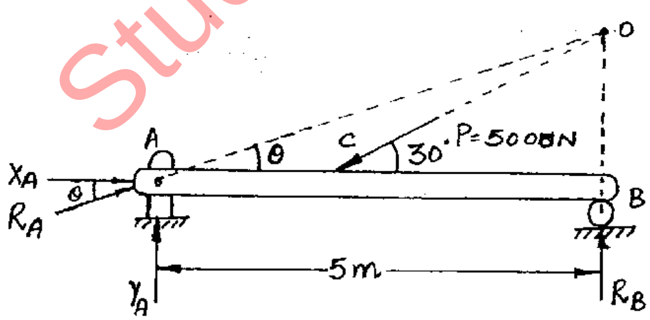
By Law of Sines :—

$$\frac{F_1}{\sin 60^\circ} = \frac{F_2}{\sin 40^\circ} = \frac{F_3}{\sin 80^\circ}$$

$$F_1 = 500 \times \frac{\sin 60^\circ}{\sin 80^\circ} = 439.6\,N.$$

$$F_2 = 500 \times \frac{\sin 40^\circ}{\sin 80^\circ} = 326.4\,N.$$

Q3.



SOL: In ΔOBC

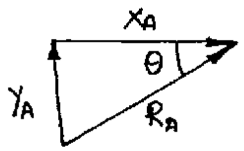
$$\frac{OB}{CB} = \tan 30^\circ \qquad \frac{OB}{AB} = \tan\theta$$

$$OB = 0.577 \times 2.5 \qquad \tan\theta = \frac{1.443}{05} = 0.2886$$

$$= 1.443\,m$$

$$\theta = \tan^{-1}(0.2886)$$

$$\boxed{\theta = 16.1^\circ}$$



Also, $\frac{Y_A}{X_A} = \tan\theta = 0.2886$

By equations of equilibrium,

$\Sigma F_x = 0$

$-P\cos 30° + X_A = 0$

$X_A = P\cos 30°$

$X_A = 5000 \times 0.866$

$\boxed{X_A = 4330\,N}$

$\Sigma F_y = 0$

$Y_A + R_B = P\sin 30°$

But, $Y_A = X_A \times \tan\theta$

$= 4330 \times 0.2886$

$\boxed{Y_A = 1250\,N}$

$R_B = 5000 \times 0.5 - Y_A$

$= 2500 - 1250$

$\boxed{R_B = 1250\,N}$

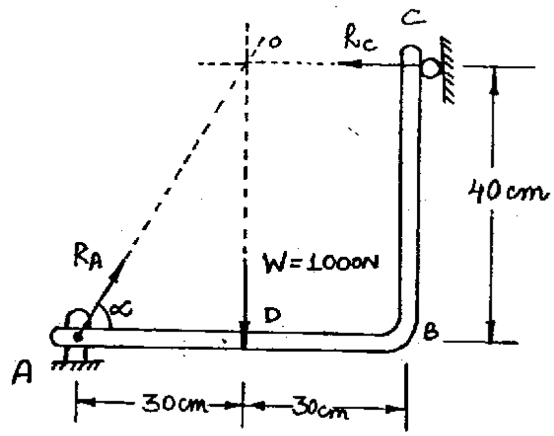
$R_A = \sqrt{X_A^2 + Y_A^2}$

$= \sqrt{(4330)^2 + (1250)^2}$

$= \sqrt{18748900 + 1562500}$

$\boxed{R_A = 4506.8\,N}$

Q4.



SOL:

By similarity of Δs.

$$\frac{R_c}{AD} = \frac{R_A}{OA} = \frac{W}{OD}$$

$$R_c = W \times \frac{AD}{OD}$$

$$= 1000 \times \frac{0.3}{0.4}$$

$$\boxed{R_c = 750N}$$

$$R_A = \frac{1000 \times OA}{0.4}$$

But $OA = \sqrt{(0.4)^2 + (0.3)^2}$

$OA = 0.5$

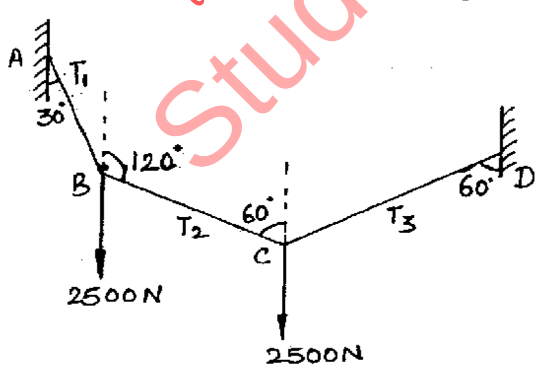
$$R_A = \frac{1000 \times 0.5}{0.4}$$

$$\boxed{R_A = 1250N}$$

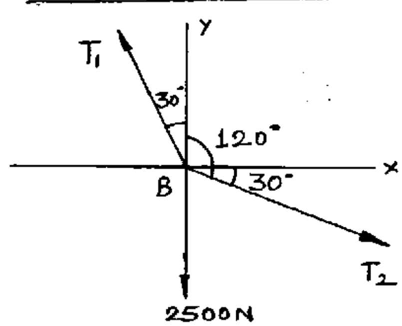
$$\tan\alpha = \frac{OD}{AD} = \frac{0.4}{0.3} = 1.333 \qquad \alpha = \tan^{-1}(1.333)$$

$$\boxed{\alpha = 53.1^\circ}$$

Q5.



Free body diagram of point B.



SOL:

$\Sigma F_x = 0$

$$T_2 \cos 30^\circ - T_1 \sin 30^\circ = 0$$

$$T_1 \sin 30^\circ = T_2 \cos 30^\circ$$

$$T_1 = T_2 \frac{\cos 30^\circ}{\sin 30^\circ} = T_2 \left(\frac{\sqrt{3}}{2}\right) \times \frac{2}{1} = \sqrt{3}\, T_2$$

$$\boxed{T_1 = \sqrt{3}\, T_2}$$

$\Sigma f_y = 0$

$$T_1 \cos 30^\circ - 2500 - T_2 \sin 30^\circ = 0$$

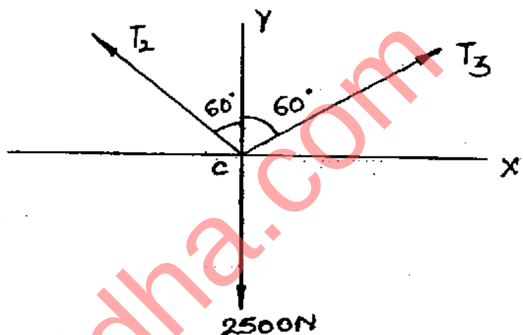
$$T_1 \times \frac{\sqrt{3}}{2} - 2500 - T_2 \times \frac{1}{2} = 0$$

$$\frac{\sqrt{3}\times\sqrt{3}}{2} T_2 - \frac{T_2}{2} = 2500 \qquad \left[\text{As } T_1 = \sqrt{3}\, T_2\right]$$

$$\boxed{T_2 = 2500N}$$

Now, $T_1 = \sqrt{3} \times 2500$

$$\boxed{T_1 = 4330N}$$



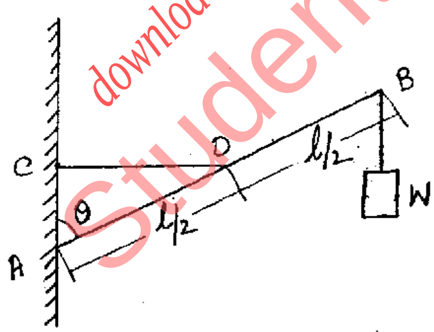
$\Sigma f_x = 0$

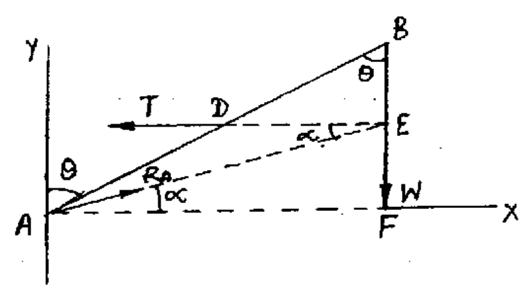
$$T_3 \cos 30^\circ - T_2 \cos 30^\circ = 0$$

$$T_3 = T_2$$

$$\boxed{T_3 = 2500N}$$

Q6.





SOL:

$$\cos\theta = \frac{BE}{BD} \Rightarrow BE = \frac{l}{2}\cos\theta = EF$$

$$\tan\alpha = \frac{EF}{AF} \qquad \left[\frac{AF}{AB} = \sin\theta, \quad AF = l\sin\theta\right]$$

$$= \frac{l \cos\theta}{2\, l \sin\theta}$$

$$\tan\alpha = \frac{l \cot\theta}{l \times 2} = \frac{1}{2}\cot\theta$$

$$\alpha = \tan^{-1}\left(\cot\theta\right)$$

Equations of equilibrium :

$\Sigma F_x = 0$

$$R_A \cos\alpha - T = 0$$

$$T = R_A \cos\alpha \quad \text{——} \; (1)$$

$\Sigma F_y = 0$

$$R_A \sin\alpha - W = 0$$

$$W = R_A \sin\alpha \quad \text{——} \; (2)$$

Dividing (1) and (2)

$$\frac{W}{T} = \frac{R_A \sin\alpha}{R_A \cos\alpha}$$

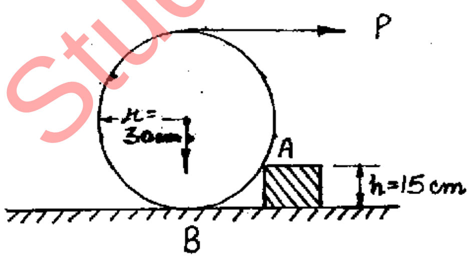
$$\tan\alpha = \frac{W}{T}$$

Also $\quad \tan\alpha = \dfrac{\cot\theta}{2}$

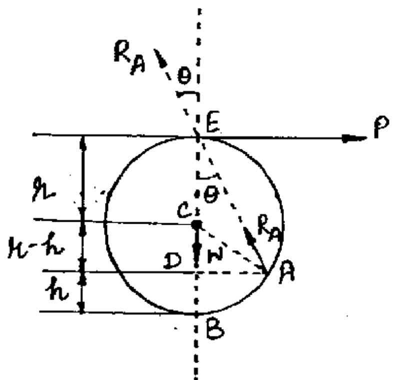
$$\therefore \quad T = \frac{2W}{\cot\theta}$$

$$\boxed{T = 2W\tan\theta}$$

Q7.



SOL:



By equations of equilibrium:

$\Sigma F_x = 0$

$$P - R_A \sin\theta = 0$$

$$R_A \sin\theta = P$$

$\Sigma F_y = 0$

$$R_A \cos\theta - W = 0$$

$$R_A \cos\theta = W$$

$$\frac{R_A \sin\theta}{R_A \cos\theta} = \frac{P}{W}$$

$$\tan\theta = \frac{P}{W}$$

In Δ ADE

$\tan\theta = \frac{AD}{DE}$

$DE = r + r - h = 2r - h$

$AD = \sqrt{(CA)^2 - (CD)^2}$

$= \sqrt{r^2 - (r-h)^2}$

$\therefore \quad \tan\theta = \frac{\sqrt{r^2-(r-h)^2}}{2r-h}$

$= \frac{\sqrt{r^2-r^2-h^2+2rh}}{2r-h}$

$= \frac{\sqrt{2rh-h^2}}{2r-h} = \frac{\sqrt{h}\,(\sqrt{2r-h})}{(2r-h)}$

$= \frac{\sqrt{h}}{\sqrt{2r-h}} = \frac{\sqrt{0.15}}{\sqrt{0.6-0.15}} = \frac{\sqrt{0.15}}{\sqrt{0.45}}$

$= \frac{0.387}{0.670}$

$\tan\theta = 0.5776$

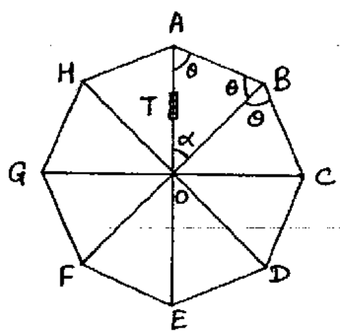
Also $\quad \tan\theta = \frac{P}{W} \quad \therefore \frac{P}{W} = 0.5776$

$P = W \times 0.5776 = \underline{\underline{577N}}$

Q8.

T= 1000N

Axial forces in bars AB and OB



SOL:

$\angle\alpha = \frac{360^\circ}{8}$

$\boxed{\angle\alpha = 45^\circ}$

$\angle\theta = \frac{135}{2}$

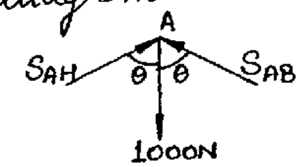
$\boxed{\angle\theta = 67.5^\circ}$

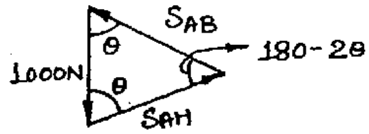
$\angle\alpha + \angle\theta + \angle\theta = 180^\circ$

$\angle\alpha + 2\angle\theta = 180^\circ$

$\angle\theta = (180 - 45)/2$

Free body diagram of point A:



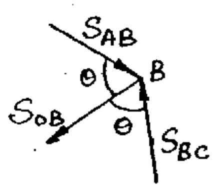


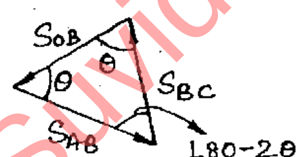
$$\frac{S_{AB}}{\sin\theta} = \frac{S_{AH}}{\sin\theta} = \frac{1000}{\sin(180-2\theta)}$$

$$S_{AB} = 1000 \times \frac{0.9238}{0.707} \qquad S_{AH} = 1000 \times \frac{0.9238}{0.707}$$

$$\boxed{S_{AB} = 1306.6\,N} \qquad \boxed{S_{AH} = 1306.6\,N}$$

Free body diagram of point B:



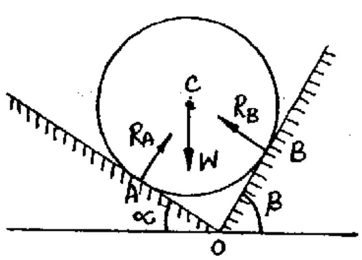


$$\frac{S_{OB}}{\sin(180-2\theta)} = \frac{S_{AB}}{\sin\theta} = \frac{S_{BC}}{\sin\theta}$$

$$S_{OB} = 1306.6 \times \frac{0.707}{0.9238}$$

$$\boxed{S_{OB} = 1000N}$$

Q9.



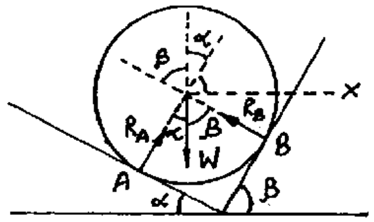
Weight $= W = 500\,N$

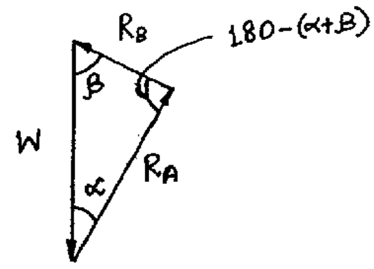
Radius $= r$

$\alpha = 25°$

$\beta = 65°$

SOL:





By law of Sines:

$$\frac{W}{\sin(180-(\alpha+\beta))} = \frac{R_B}{\sin\alpha} = \frac{R_A}{\sin\beta}$$

$$\frac{W}{\sin 90^\circ} = \frac{R_B}{\sin 25^\circ} = \frac{R_A}{\sin 65^\circ}$$

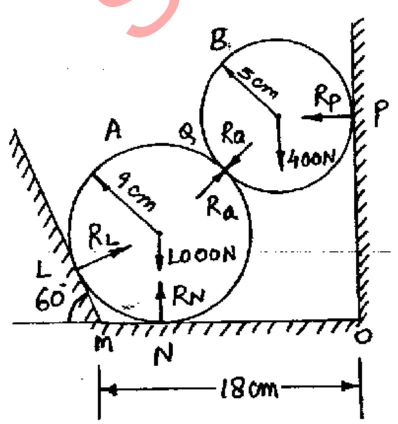
$$R_B = 500 \times \frac{\sin 25^\circ}{\sin 90^\circ} = 500 \times \frac{0.4226}{1}$$

$$\boxed{R_B = 211.3\,N}$$

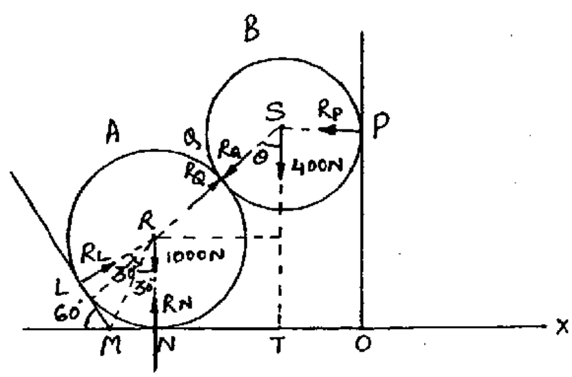
$$R_A = 500 \times \frac{\sin 65^\circ}{\sin 90^\circ} = 500 \times \frac{0.9063}{1}$$

$$\boxed{R_A = 453.15\,N}$$

Q10.



Free body diagram of cylinder A and B.



M, R and S should be

In ΔRSU

$$\sin\theta = \frac{RU}{RS}$$

Now, $RS = RQ + QS$

$= 9 + 5 = 14 \text{ cm}$

$RU = NT = MO - TO - MN$

$= 18 - MN - 5$

$= 13 - MN$

$= 13 - 5.2$

$RU = 7.8 \text{ cm}.$

$\left[\begin{aligned} MN &= RN \tan 30^\circ \\ &= 9 \times 0.577 \\ &= 5.2 \text{ cm} \end{aligned} \right]$

$\therefore \quad \sin\theta = \frac{7.8}{14} = 0.5571$

$$\boxed{\theta = 33.86^\circ}$$

Equations of equilibrium for cylinder B

$\Sigma F_x = 0$

$-R_P + R_Q \sin 33.86^\circ = 0$

$\Sigma F_y = 0$

$R_Q \cos 33.86^\circ - 400 = 0$

$$R_Q = \frac{400}{\cos 33.86^\circ}$$

$$\boxed{R_Q = 481.69 \text{ N}}$$

$R_P = 481.69 \times \sin 33.86^\circ$

$= 481.69 \times 0.5571$

$$\boxed{R_P = 268.38 \text{ N}}$$

Equations of equilibrium for cylinder A

$\Sigma F_x = 0$

$$R_L \sin 60^\circ - R_Q \sin 33.86^\circ = 0$$

$$R_L = \frac{R_Q \sin 33.86}{\sin 60^\circ}$$

$$= 481.69 \times \frac{0.557}{0.866}$$

$$\boxed{R_L = 310N}$$

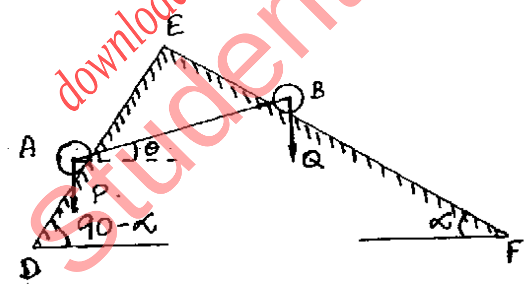
$\Sigma F_y = 0$

$$R_N - 1000 - R_Q \cos 33.86^\circ + R_L \cos 60^\circ = 0$$

$$R_N = 1000 + 399.99 - 155$$

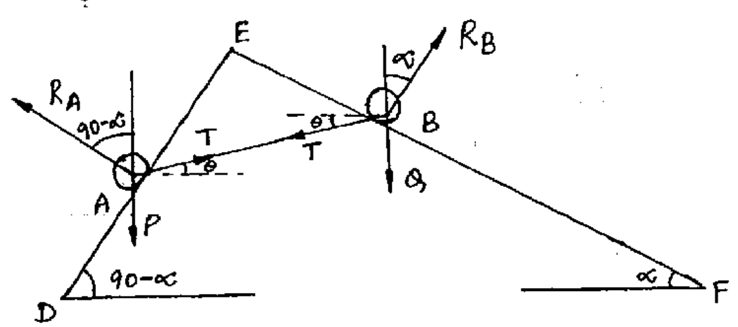
$$\boxed{R_N = 1245\,N}$$

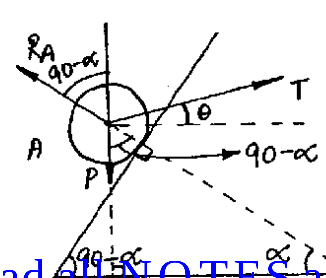
Q11.

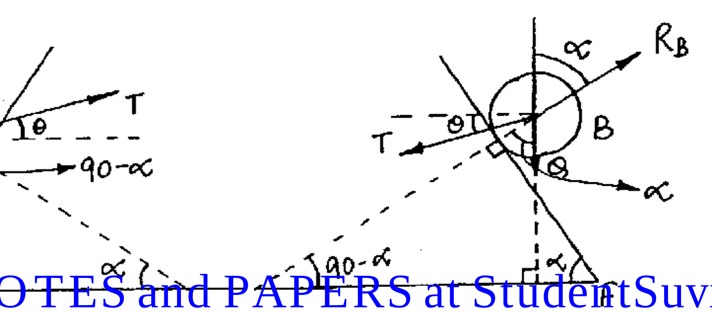


$P = 50N$
$Q = 100N$
$\alpha = 30^\circ$

Sol:







By equations of equilibrium for roller P.

$\Sigma F_x = 0$

$T\cos\theta - R_P \sin(90-\alpha) = 0$

$T\cos\theta = R_P \sin(90-\alpha)$

$T\cos\theta = R_P \cos\alpha \longrightarrow (1)$

$\Sigma F_y = 0$

$R_P \cos(90-\alpha) + T\sin\theta - P = 0$

$R_P \sin\alpha + T\sin\theta - P = 0 \longrightarrow (2)$

Equations of equilibrium for roller Q.

$\Sigma F_x = 0$

$R_Q \sin\alpha - T\cos\theta = 0$

$R_Q \sin\alpha = T\cos\theta \longrightarrow (3)$

$\Sigma F_y = 0$

$R_Q \cos\alpha - T\sin\theta - Q = 0 \longrightarrow (4)$

Using (1)

$$R_P = T\frac{\cos\theta}{\cos\alpha}$$

Substitute in equation (2)

$$T\frac{\cos\theta}{\cos\alpha} \times \sin\alpha + T\sin\theta - P = 0$$

$P = T(\sin\theta + \cos\theta\tan\alpha) \longrightarrow (5)$

Using (3)

$$R_Q = \frac{T\cos\theta}{\sin\alpha}$$

Substitute in equation (4)

$$T\frac{\cos\theta}{\sin\alpha}\cos\alpha - T\sin\theta - Q = 0$$

$$Q = T(\cos\theta\cot\alpha - \sin\theta) \longrightarrow (6)$$

Dividing (5) by (6)

$$\frac{Q}{P} = \frac{T(\cos\theta\cot\alpha - \sin\theta)}{T(\sin\theta + \cos\theta\tan\alpha)}$$

$$Q(\sin\theta + \cos\theta\tan\alpha) = P(\cos\theta\cot\alpha - \sin\theta)$$

Dividing by $\cos\theta$.

$$Q(\tan\theta + \tan\alpha) = P(\cot\alpha - \tan\theta)$$

$$Q\tan\theta + Q\tan\alpha = P\cot\alpha - P\tan\theta$$

$$\tan\theta(P+Q) = P\cot\alpha - Q\tan\alpha$$

$$\tan\theta = \frac{P\cot\alpha - Q\tan\alpha}{P+Q}$$

We Know,

$$P = 50N, \; Q = 100N, \; \alpha = 30^\circ$$

$$\tan\theta = \frac{50\cot 30^\circ - 100\tan 30^\circ}{50+100}$$

$$= \frac{86.60 - 57.73}{150}$$

$$\tan\theta = 0.1924$$

$$\boxed{\theta = 10.9^\circ}$$

Using equation (6)

$$T = \frac{Q}{\cos\theta \cot\alpha - \sin\theta}$$

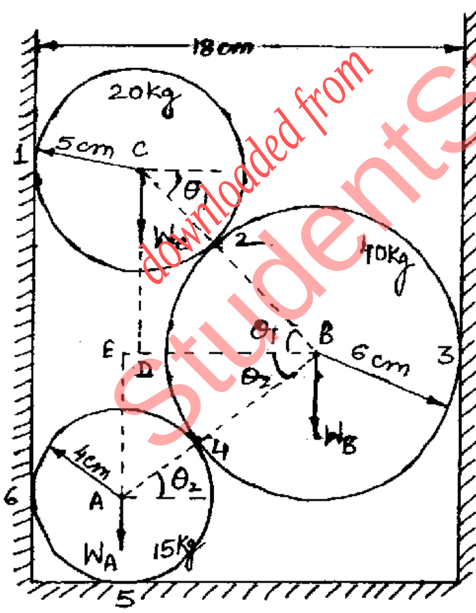
$$= \frac{100}{\cos 10{\cdot}9 \cot 30^\circ - \sin 30^\circ}$$

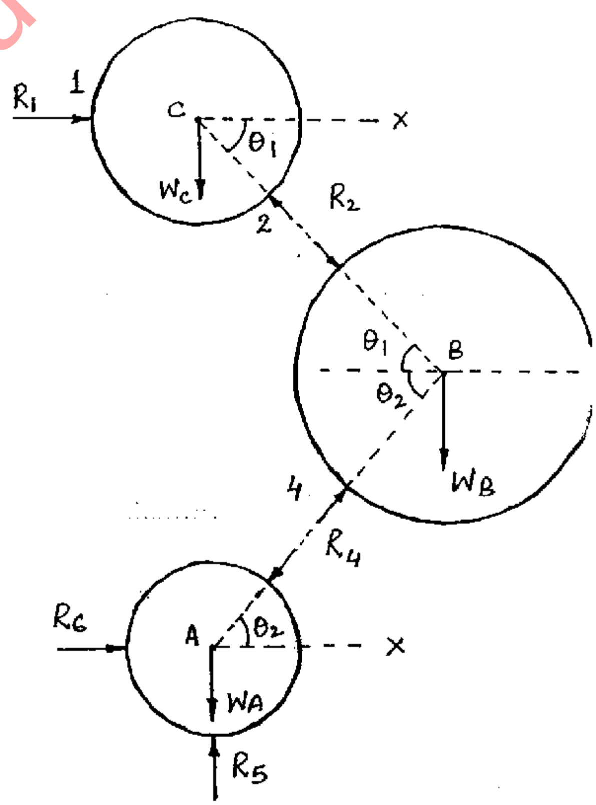
$$= \frac{100}{1{\cdot}7 - 0{\cdot}5}$$

$$= \frac{100}{1{\cdot}2}$$

$$\boxed{T = 83{\cdot}33\,N}$$

Q12.





In ΔBCD

$$\cos\theta_1 = \frac{BD}{BC} = \frac{18-5-6}{6+5}$$

$$= \frac{7}{11}$$

$$\cos\theta_1 = 0.636$$

$$\theta_1 = 50.47^\circ$$

$$\sin\theta_1 = 0.771$$

In ΔABE

$$\cos\theta_2 = \frac{BE}{AB} = \frac{18-6-4}{6+4}$$

$$= \frac{8}{10}$$

$$\cos\theta_2 = 0.8$$

$$\theta_2 = 36.86^\circ$$

$$\sin\theta_2 = 0.6$$

Equations of equilibrium for cylinder C

$$\Sigma F_x = 0$$

$$R_1 - R_2\cos\theta_1 = 0$$

$$R_1 = R_2\cos\theta_1$$

$$R_1 = 0.636\ R_2 \qquad \boxed{R_1 = 161.85\,N}$$

$$\Sigma F_y = 0$$

$$R_2\sin\theta_1 - W_c = 0$$

$$W_c = R_2\sin\theta_1$$

$$R_2 = \frac{20 \times 9.81}{0.771}$$

$$\boxed{R_2 = 254.5\,N}$$

Equations of equilibrium for cylinder B

$$\Sigma F_x = 0$$

$$R_4\cos\theta_2 - R_3 + R_2\cos\theta_1 = 0$$

$$0.8R_4 - R_3 + 161.86 = 0$$

$$\boxed{R_3 = 94666\,N}$$

$$\Sigma F_y = 0$$

$$R_4\sin\theta_2 - R_2\sin\theta_1 - W_B = 0$$

$$0.6R_4 - 196.22 - 392.4 = 0$$

$$R_4 = \frac{588.62}{0.6} \Rightarrow \boxed{R_4 = 981\,N}$$

Equations of equilibrium for cylinder A.

$\Sigma F_x = 0$

$$R_6 - R_4 \cos\theta_2 = 0$$

$$R_6 = R_4 \cos\theta_2$$

$$\boxed{R_6 = 784.8 N}$$

$\Sigma F_y = 0$

$$R_5 - R_4 \sin\theta_2 - W_A = 0$$

$$R_5 - 588.6 - 147.15 = 0$$

$$\boxed{R_5 = 735.75 N}$$

MOMENT OF A FORCE

It is defined as the measure of the turning effect produced by a force on a body.

Thus, besides producing translatory motion, a force can produce rotary motion as well.

MOMENT OF A FORCE ABOUT AN AXIS.

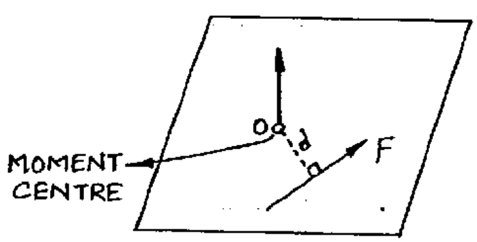
The moment of a force about a point, is equal to the product of the force and perpendicular distance of the point from the line of action of the force.

$$M_o = F \times d$$

Point O $\longrightarrow$ Moment centre

Distance d $\longrightarrow$ Arm of the force.

Unit of moment = N-m

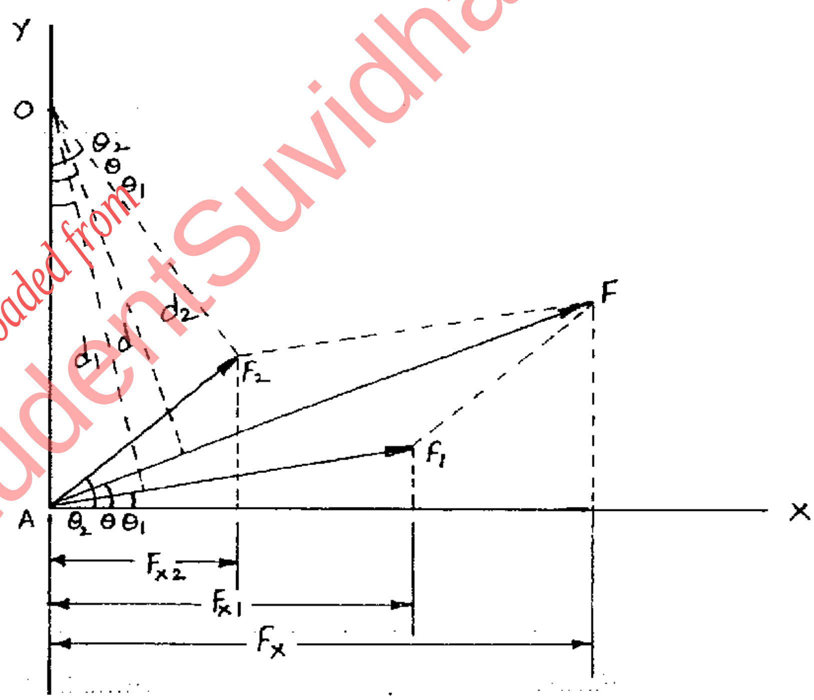


1) An anticlockwise moment is taken positive.
2) Clockwise moment is taken negative.

THEOREM OF VARIGNON

"The moment of a force about an axis is equal to the sum of the moments of its components about the same axis."

Let a force F act at a point A and it has two components F_1 and F_2 in any two directions.
Let O point be moment centre.



Moment of the force F about O

$$Fd = F(OA\cos\theta) = OA\,(F\cos\theta)$$

$$Fd = OA\,F_x \longrightarrow (1)$$

Moment of the force F_1 about O

$$F_1 d_1 = F_1(OA\cos\theta_1) = OA\,(F_1\cos\theta_1)$$

$$F_1 d_1 = OA\,F_{x_1} \longrightarrow (2)$$

Moment of the force F_2 about O

$$F_2 d_2 = F_2(OA \cos\theta_2) = OA(F_2 \cos\theta_2)$$

$$F_2 d_2 = OA\, F_{x_2} \longrightarrow (3)$$

Adding (3) and (2)

$$F_1 d_1 + F_2 d_2 = OA\, F_{x_1} + OA\, F_{x_2}$$

$$= OA(F_{x_1} + F_{x_2}) \longrightarrow (4)$$

But $\quad F_x = F_{x_1} + F_{x_2}$

Sum of x-components of forces F_1 and F_2 = x-component of the resultant force F.

$$OA(F_x) = OA(F_{x_1} + F_{x_2})$$

From (1) and (4)

$$\boxed{Fd = F_1 d_1 + F_2 d_2}$$

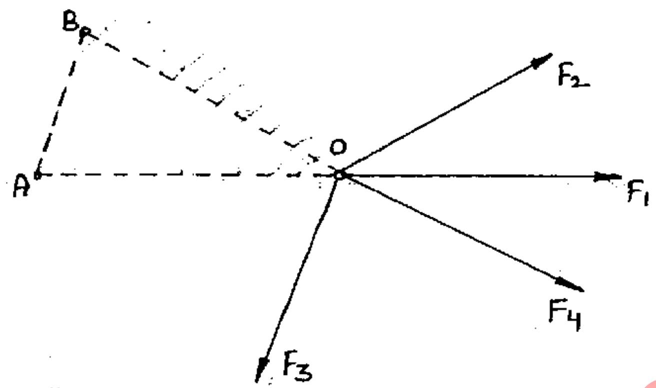
EQUATIONS OF EQULIBRIUM

Let the sum of the moments of coplanar concurrent forces be zero. There exists two conditions:

1) Resultant of the system of forces is zero and forces are in equilibrium.

2) Moment centre lies on the line of action of the resultant.

Consider a system of four forces F_1, F_2, F_3, F_4 acting at a point O.



Let Algebraic sum of moments of these forces about a point A be zero

$$\Sigma M_A = 0$$

then, a) Resultant of forces F_1, F_2, F_3, F_4 is zero.

b) Point A lies on the line of action of the resultant.

Now, take another point B and find sum of moments of these forces about point B.

Let

$$\Sigma M_B = 0$$

then,

a) Resultant force is zero.

b) Line of action of the resultant lies along OB.

We know,

the resultant cannot have two lines of action i.e along OA and OB, therefore resultant must be zero.

Therefore,

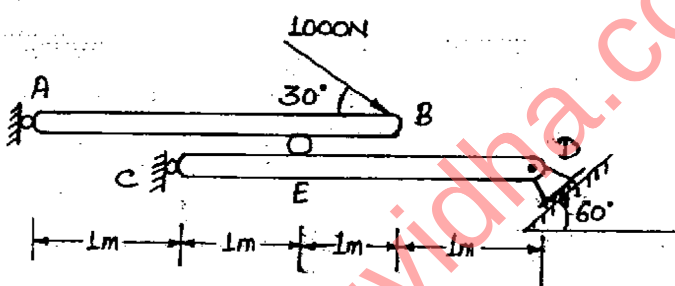
$$\Sigma M_A = 0$$

$$\Sigma M_B = 0$$

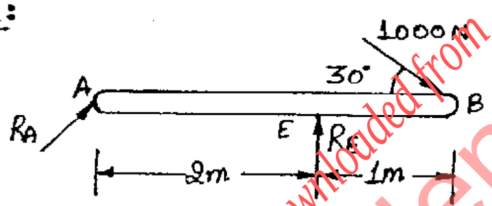
These two equations are known as "Moment Equations of Equilibrium."

EXAMPLES

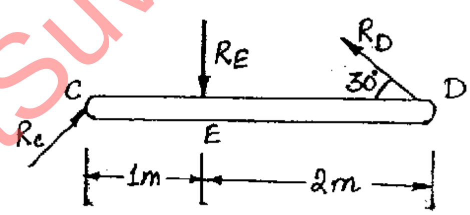
Q1.



SOL:



Free body diagram of beam AB.



Free body diagram of beam CD

Taking moments about A

$$\Sigma M_A = 0$$

$$R_E \times 2 - 1000 \times 3 \times \sin 30^\circ = 0$$

$$2R_E = 500 \times 3$$

$$\boxed{R_E = 750N}$$

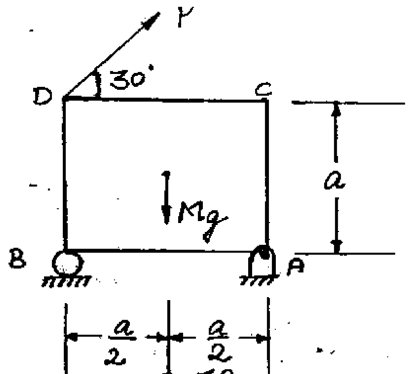
Taking moment about C.

$$\Sigma M_c = 0$$

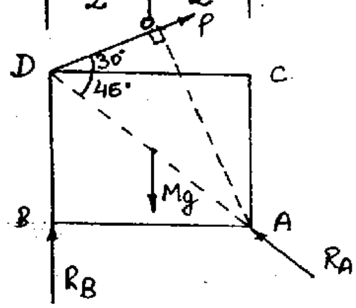
$$-R_E + R_D \, 3 \sin 30^\circ = 0$$

$$R_D = \frac{750}{3 \times 0.5} = 500 \qquad \boxed{R_D = 500N}$$

Q2.



SOL :



$R_B = 0$ because as the block is lifted off the roller B, no contact remains with the roller.

$\therefore$ Taking moment about A

$\Sigma M_A = 0$

$$Mg\left(\frac{a}{2}\right) - P(AD \sin 75^\circ) = 0 \qquad [OA = AD \sin 75^\circ]$$

Now $AD = \sqrt{(CD)^2 + (AC)^2}$

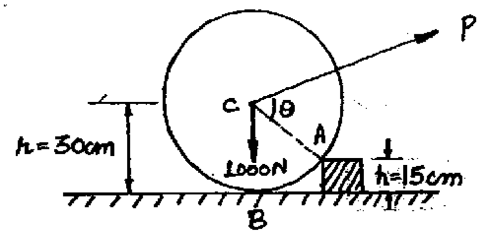
$$= \sqrt{a^2 + a^2} = a\sqrt{2}$$

$$\therefore \quad Mg\frac{a}{2} = P(a\sqrt{2}\sin 75^\circ)$$

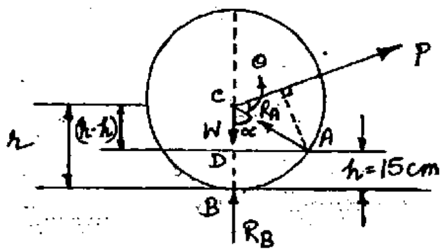
$$P = \frac{Mg \times a}{2\sqrt{2}\, a \sin 75^\circ}$$

$$\boxed{P = 0.366\, Mg}$$

Q3.



SOL:



$R_B = 0$ because as the wheel is about to turn, there is no contact between the point B and the wheel.

$\therefore$ Taking moment about A

$\Sigma M_A = 0$

$W(AD) - P(AC \sin\theta) = 0$

$P(AC \sin\theta) = W(AD)$

$P = \dfrac{W(AD)}{AC \sin\theta}$

$AD = \sqrt{AC^2 - CD^2}$ $\qquad [CD = r - h]$

$= \sqrt{r^2 - (r-h)^2}$

$= \sqrt{(r+r-h)(r-r+h)}$

$= \sqrt{(2r-h)h}$

$AD = \sqrt{2rh - h^2}$

$\therefore \quad P = \dfrac{W\sqrt{2rh-h^2}}{r \sin\theta}$

For P to be minimum, $\sin\theta$ should be maximum

$$\sin\theta = 1$$

$$\boxed{\theta = 90^\circ}$$

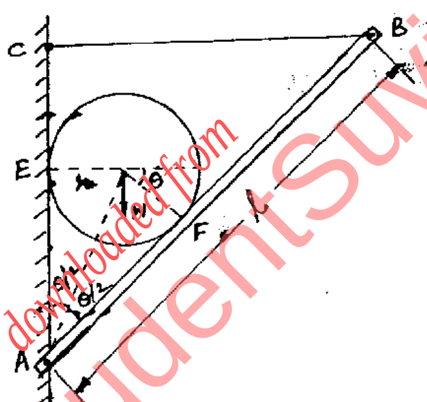
P acts at right angles to AC

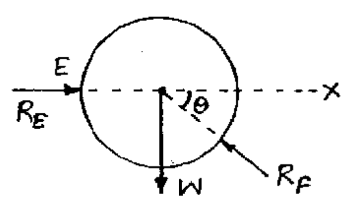
$$P_{min} = \frac{1000\sqrt{2\times 0.3\times 0.15 - 0.15\times 0.15}}{0.3\ \sin 90^\circ}$$

$$= \frac{1000\sqrt{0.09 - 0.0225}}{0.3}$$

$$\boxed{P_{min} = 866\,N}$$

Q4.





SOL:

Equation of equilibrium

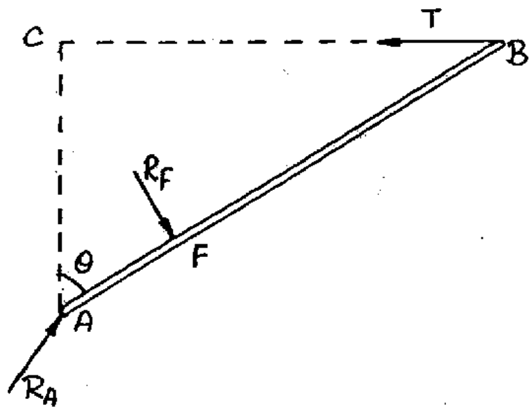
$$\Sigma F_y = 0$$

$$R_F \sin\theta - W = 0$$

$$R_F = \frac{W}{\sin\theta}$$

Reaction on cylinder = Reaction of bar = R_F
[Direction is opposite]

Consider free body diagram of bar



Taking moment about A

$$\Sigma M_A = 0$$

$$T \cdot AC - R_F \cdot AF = 0$$

In ΔABC

$$AC = l \cos\theta \longrightarrow (1)$$

In ΔAOF

$$AF = FO \cot\frac{\theta}{2}$$

$$AF = r \cot\frac{\theta}{2} \longrightarrow (2)$$

From (1) & (2)

$$T \cdot l\cos\theta - R_F \cdot r\cot\frac{\theta}{2} = 0$$

$$T = \frac{R_F \; r\cot\frac{\theta}{2}}{l\cos\theta}$$

$$T = \frac{W}{\sin\theta} \times \frac{r\cot\theta/2}{l\cos\theta}$$

$$= \frac{W r \cot\theta/2}{2\sin\theta/2 \cos\theta/2 \times l\cos\theta}$$

$$= \frac{W r}{2 l \sin^2\frac{\theta}{2} \cos\theta}$$

Bar is vertical when $\theta = 0^\circ$ and bar is horizontal when $\theta = 90^\circ$.

Now,

Tension T to be minimum

$$\frac{dT}{d\theta} = 0$$

$$\frac{d\left[\frac{Wr}{2l\cos\theta \sin^2\theta/2}\right]}{d\theta} = 0$$

$$\sin\theta\left[4\sin^2\frac{\theta}{2} - 1\right] = 0$$

$$4\sin^2\frac{\theta}{2} = 1 \qquad [\sin\theta \neq 0]$$

$$\sin^2\frac{\theta}{2} = \frac{1}{4}$$

$$\sin\frac{\theta}{2} = \frac{1}{2} \implies \frac{\theta}{2} = 30^\circ$$

$$\boxed{\theta = 60^\circ}$$

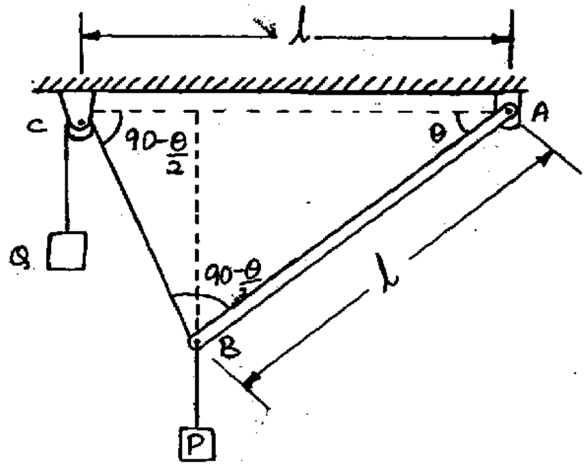
Now,

$$T = \frac{Wr}{2l\cos 60^\circ \sin^2 30^\circ}$$

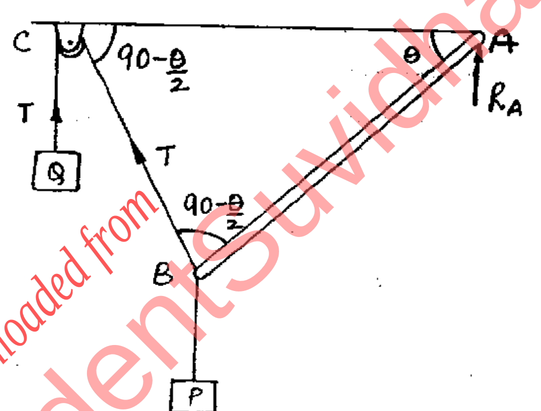
$$= \frac{Wr}{2l \times 0.25 \times 0.5}$$

$$\boxed{T = \frac{4Wr}{l}}$$

Q5.



SOL:



Taking moment about A

$$\Sigma M_A = 0$$

$$Pl\cos\theta - Ql\sin\left(90-\frac{\theta}{2}\right) = 0$$

$$P\cos\theta - Q\sin\left(90-\frac{\theta}{2}\right) = 0$$

$$P\cos\theta - Q\cos\frac{\theta}{2} = 0$$

$$P\cos\theta = Q\cos\frac{\theta}{2}$$

$$P\left[2\cos^2\frac{\theta}{2} - 1\right] - Q\cos\frac{\theta}{2} = 0$$

$$2P\cos^2\frac{\theta}{2} - P - Q\cos\frac{\theta}{2} = 0$$

$$2P\cos^2\frac{\theta}{2} - Q\cos\frac{\theta}{2} - P = 0$$

Now,

$$\cos\frac{\theta}{2} = \frac{Q \pm \sqrt{Q^2 + 8P^2}}{4P}$$

$$= \frac{1}{4}\left[\frac{Q}{P} \pm \sqrt{\frac{Q^2}{P^2} + 8}\right]$$

Given $Q = \frac{P}{2}$

$$\Rightarrow \frac{Q}{P} = \frac{1}{2}$$

$$\therefore \cos\frac{\theta}{2} = \frac{1}{4}\left[\frac{1}{2} \pm \sqrt{\frac{1}{4} + 8}\right]$$

$$= \frac{1}{4}\left[\frac{1}{2} \pm \sqrt{\frac{33}{4}}\right]$$

$$= \frac{1}{4}\left[0.5 + 2.872\right]$$

$$\cos\frac{\theta}{2} = 0.843$$

$$\frac{\theta}{2} = 32.53^\circ$$

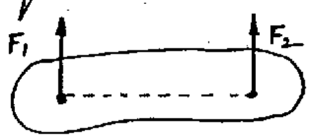
$$\boxed{\theta = 65^\circ}$$

PARALLEL FORCES IN A PLANE

TYPES OF PARALLEL FORCES

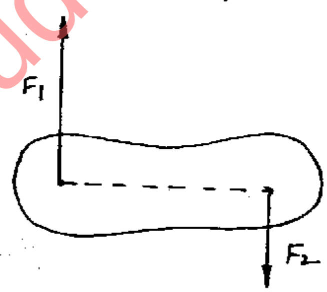
1.) LIKE PARALLEL FORCES :

Two forces parallel in nature and acting in the same direction are known as like parallel forces. These forces can be equal or unequal.



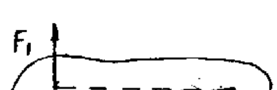
2.) UNLIKE UNEQUAL PARALLEL FORCES:

Two forces when acting in opposite directions and are in unequal magnitude are called unlike unequal parallel forces.



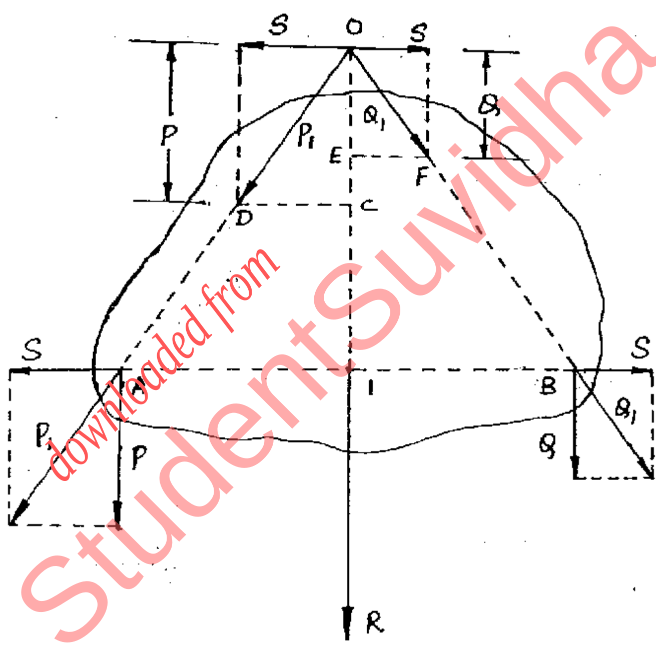
3.) UNLIKE EQUAL PARALLEL FORCES:

Two forces when acting in opposite directions but having equal magnitude are known as unlike equal parallel forces.



RESULTANT OF TWO PARALLEL FORCES ACTING IN THE SAME DIRECTION

1. Let two forces P and Q act at the points A and B of a body. Join points A and B.
2. Superimpose two equal and opposite forces at points A and B each equal to S.
3. Using parallelogram law, P_1 be resultant of P and S and Q_1 be resultant of Q and S.



4. Two parallel forces are replaced by two equivalent non-parallel forces P_1 and Q_1
5. By transmitting the points of application of the forces P_1 and Q_1 to point O, resolve the forces P_1 and Q_1 into original components.
6. The components S at O of P_1 and Q_1 cancel as they are collinear and opposite in direction.
7. Other components P and Q act along same line OI and in same direction, hence resultant is given by

$$R = P + Q.$$

→ Resultant of two like parallel forces is equal to sum of the two parallel forces and it acts in a direction parallel to them.

Line of action of Resultant

Consider ΔOAI and ODC

$$\frac{OI}{OC} = \frac{AI}{CD} = \frac{OA}{OD}$$

Now,

$$\frac{OI}{IA} = \frac{OC}{CD} = \frac{P}{S} \longrightarrow (1)$$

Similarly from ΔOBI and OEF

$$\frac{OI}{IB} = \frac{OE}{EF} = \frac{Q}{S} \longrightarrow (2)$$

From (1) and (2)

$$\frac{\frac{OI}{IA}}{\frac{OI}{IB}} = \frac{\frac{P}{S}}{\frac{Q}{S}}$$

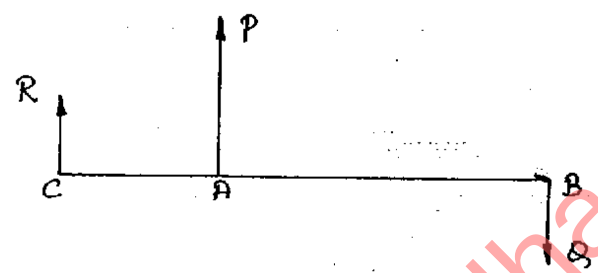
$$\boxed{\frac{IB}{IA} = \frac{P}{Q}}$$

→ Resultant of two like parallel forces acts parallel to them and its position is such that it divides the distance between their points of application in the ratio which is inversely proportional to their magnitude.

RESULTANT OF TWO UNEQUAL PARALLEL FORCES ACTING IN OPPOSITE DIRECTIONS

Two unequal forces are acting at points A and B let force P is larger than Q.

If R is the resultant of P and Q acting at a point C, then



Point C lies outside of AB and on same side as the larger force P. The position of point C is such that

$$\boxed{\frac{P}{Q} = \frac{BC}{AC}}$$

→ Resultant of two unequal parallel forces lies outside the line joining the points of action of the two forces and on the same side as the large force.

TWO EQUAL PARALLEL FORCES ACTING IN OPPOSITE DIRECTIONS : COUPLE

Two equal parallel forces acting in opposite directions can never be replaced by a single force. Such forces form a couple which tend to rotate the body.

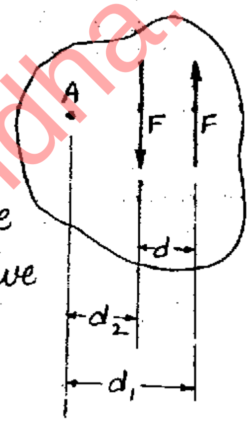
ARM OF THE COUPLE: Perpendicular distance between the lines of action of the two forces.

MOMENT OF A COUPLE: Rotational tendency of couple is measured by the moment.

It is the product of either of the forces forming the couple and the arm of the couple.

Moment of couple

$= F \times d$



Anticlockwise moment → Positive

Clockwise moment → Negative

NOTE:

1) Algebraic sum of the forces forming a couple is zero.

2) Algebraic sum of the moments of two forces forming a couple is independent of position of the moment centre.

Taking moment about A

$$M_A = F.d_1 - Fd_2$$

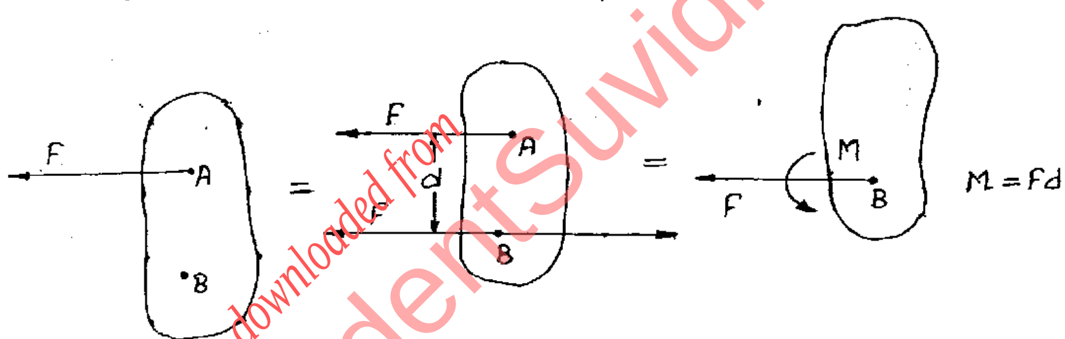
$$\boxed{M_A = Fd}$$

3) For adding the couple, take algebraic sum of their moments.

Hence, two couples acting in a plane can be in equilibrium if their moments are equal in magnitude and opposite in direction.

RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

Consider a force F acting at a point A on a body. It is to be replaced by a force acting at a point B and a couple.



At point B introduce two forces, each of magnitude F and acting parallel to the force at point A.

From these three forces take two forces acting in opposite directions at points A and B. They form a couple

$$\boxed{M = Fd}$$

Hence, original force F acting at point A is replaced by a force F at point B together with a couple of magnitude F×d.

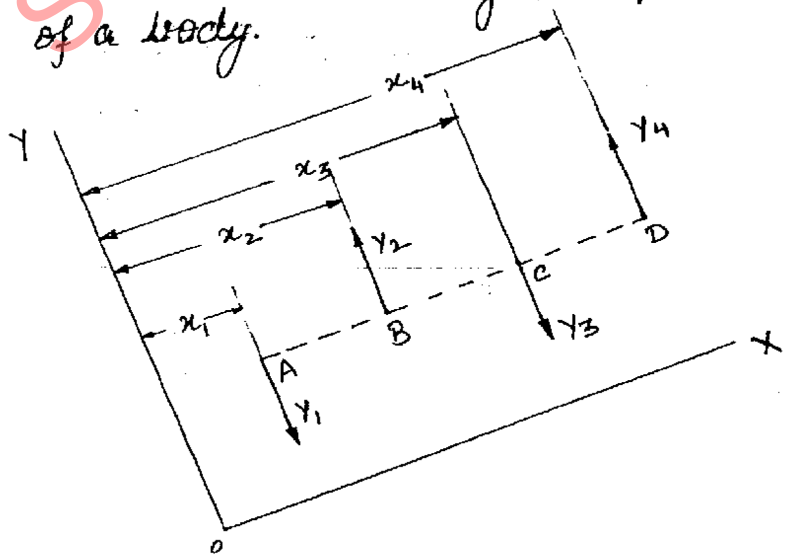
EQUIVALENT SYSTEM OF FORCES

→ Operations by which one system of forces can be transformed into equivalent system of forces:

1) Replacing of two forces acting at a point by a resultant.
2) Resolving a force into two components.
3) Cancelling two equal and opposite forces acting at a point.
4) Attaching two equal and opposite forces at a point.
5) Transmitting a force along its line of action.

GENERAL CASE OF PARALLEL FORCES IN A PLANE

Consider number of forces parallel and coplanar in nature Y_1, Y_2, Y_3, Y_4 acting at points A, B, C and D of a body.



Taking y-axis parallel to these forces and x-axis perpendicular to them.

Let the distances of forces from y-axis be x_1, x_2, x_3, x_4 for forces Y_1, Y_2, Y_3, Y_4 respectively.

Similarly the resultant force Y lies at a distance x from y-axis.

$$Y = Y_1 + Y_2 + Y_3 + Y_4 \longrightarrow$$ After resolving along y-axis.

$$\boxed{Y = \Sigma Y_i} \longrightarrow (1)$$

Now sum of moments of all these forces about the origin O

$$M_0 = Y_1 x_1 + Y_2 x_2 + Y_3 x_3 + Y_4 x_4$$

$$\boxed{M_0 = \Sigma Y_i x_i} \longrightarrow (2)$$

CASE I

The system of forces reduces to a single resultant force Y.

Magnitude of Resultant $= Y = \Sigma Y_i$

Moment of Resultant $= Yx \longrightarrow (3)$

Equating (2) and (3)

$$M_0 = Yx$$

$$x = \frac{M_0}{Y} = \frac{\Sigma(Y_i x_i)}{Y} = \frac{\Sigma(Y_i x_i)}{\Sigma Y_i}$$

$$\boxed{x = \frac{\Sigma(Y_i x_i)}{\Sigma Y_i}}$$

CASE II

The system of forces reduces to a couple.

Couple is formed by two equal and opposite parallel forces. Hence, resultant of all the forces must be zero.

$$\boxed{Y = \Sigma Y_i = 0}$$

Magnitude of moment of couple.

$$\boxed{M_o = \Sigma Y_i x_i}$$

CASE III

The system is in equilibrium

$$Y = 0 \Longrightarrow \Sigma Y_i = 0$$
$$M_o = 0 \Longrightarrow \Sigma Y_i x_i = 0$$

→ Condition of equilibrium can also be expressed by two moment equations:

$$\Sigma (M_A)_i = 0$$
$$\Sigma (M_B)_i = 0$$